N92-10883

MAPPING THE STABILITY FIELD OF JUPITER TROJANS. H.F. Levison, U.S. Naval Observatory, Flagstaff, AZ 86002, E.M. Shoemaker and R.F. Wolfe, U.S. Geological Survey, Flagstaff, AZ 86001

Jupiter Trojans are a remnant of outer solar system planetesimals captured into stable or quasistable libration about the 1 : 1 resonance with the mean motion of Jupiter. The observed swarms of Trojans may provide insight into the original mass of condensed solids in the zone from which the Jovian planets accumulated, provided that the mechanisms of capture can be understood [1]. As a first step toward this understanding, we have undertaken to map the stability field of Trojans in the coordinates proper eccentricity, e_p, and libration amplitude, D. To accomplish this mapping, we integrated numerically the orbits of 110 particles with e_p in the range 0 to 0.8 and D in the range 0° to 140°. Orbits of the Sun, the four Jovian planets, and the massless particles were integrated as a full N-body system, in a barycentric frame using a fourth order symplectic scheme [2, 3]. Initial positions and velocities for the planets were taken from [4].

Our definitions of e_p and D for Trojans are an extension of those presented in [1]. We utilized the analytic work of Érdi [5] to obtain

$$e_p^2 = [e\cos\tilde{\omega} - e_j\cos(\tilde{\omega}_j \pm \tilde{\omega}_b)]^2 + [e\sin\tilde{\omega} - e_j\sin(\tilde{\omega}_j \pm \tilde{\omega}_a)]^2,$$

where e is the osculating eccentricity, $\tilde{\omega}$ is the longitude of perihelion,

$$\tilde{\omega}_a = \sin^{-1}\left(\frac{\sqrt{3}}{2} + \frac{73\sqrt{3}}{2^4 3^2}l^2 + \frac{6233\sqrt{3}}{2^8 3^4}l^4\right),\quad \tilde{\omega}_b = \cos^{-1}\left(\frac{1}{2} + \frac{17}{2^4 3}l^2 + \frac{329}{2^8 3^3}l^4\right),$$

and the subscript j refers to elements of Jupiter. The parameter $l \approx 8.58 \times 10^{-3}D$, a result not explicitly given by Érdi.

In our study of highly eccentric orbits we found that the longitude of the libration point is not fixed at ±60° from the mean longitude of Jupiter but is a function of e_p. This is illustrated by inserts in Fig. 1, that show the trajectories of three particles in the rotating frame. The trajectories illustrated correspond to small values of D; e_p = 0.03, 0.3, and 0.7 in frames A, B, and C respectively. Small circles in Fig. 1 represent measured values of the angle between the libration point and the mean longitude of Jupiter, ϕ_L, as a function of e_p; the solid curve is a fourth order polynomial fit to the observations,

$$\phi_L = 1.047 + 1.56e_p + 0.901e_p^2 + 3.39e_p^3 - 3.71e_p^4.$$

We calculate D as follows: Let $x = |\sin((\phi - \phi_L + \pi/3)/2)|$, then

$$E_{eq} = -\frac{1}{6}\left(\frac{\partial\phi}{\partial t}\right)^2 - \frac{m\eta_j^2}{2x}\left[1 + 4x^3\right],$$

where $\frac{\partial\phi}{\partial t} = \eta - \eta_j$, ϕ is the difference in mean longitude between the Trojan and Jupiter, m is the mass of Jupiter in solar masses, and η is the mean motion of the object. Solutions for the minimum and maximum values of ϕ can be found [6] from $x_{min} = \sin(\alpha/3)/3A^{1/2}$ and $x_{max} = \sin(\alpha/3 + 120°)/3A^{1/2}$, where $A = -m\eta_j^2/2E_{eq}$ and $\sin\alpha = (3A)^{3/2}$. The libration amplitude D is $\phi_{max} - \phi_{min}$.

We integrated the orbits for time intervals corresponding to 1 500, 15 000, and 150 000 Jupiter orbital periods. Test particles were started with orbits in Jupiter's orbital plane. In a preliminary series of integrations carried out to 15 000 orbital periods, where the initial inclination of the orbits was 10°, we found no significant difference in the limit of the stability field from that obtained with $i = 0$.

The distribution of orbits that are stable over the different intervals of time that we investigated is illustrated in Fig. 2, where the limits of stability for time intervals of 17 800, 178 000, and 1 780 000 years are contoured. Orbits with surprisingly high initial e_p are stable for nearly 2 million years, but no orbits with D exceeding about 110° are stable for more than 17 800 years. With increasing time the stability field shrinks in both e_p and D. The limit of the main field of observed Trojan asteroids shown in Fig. 2 may represent the approximate limit of stability for a time interval of 4.5 billion years.

One recently discovered Trojan, 1989 BQ, lies well outside the main Trojan field ($e_p = 0.22$, $D = 17°$). We suggest that the dynamical lifetime of 1989 BQ may be of order 10^8 years and that it has been captured late in solar system history. Capture may have occurred by diffusion, successively, from a free orbit to a horseshoe orbit to tadpole orbits of increasing stability. If one Trojan bright enough to be discovered by the usual methods of asteroid search has been captured late in solar system time, most of the known Trojans may have been captured by a similar process at an early time when the flux of Jupiter-crossing bodies was many orders of magnitude higher than at present [1].

References:

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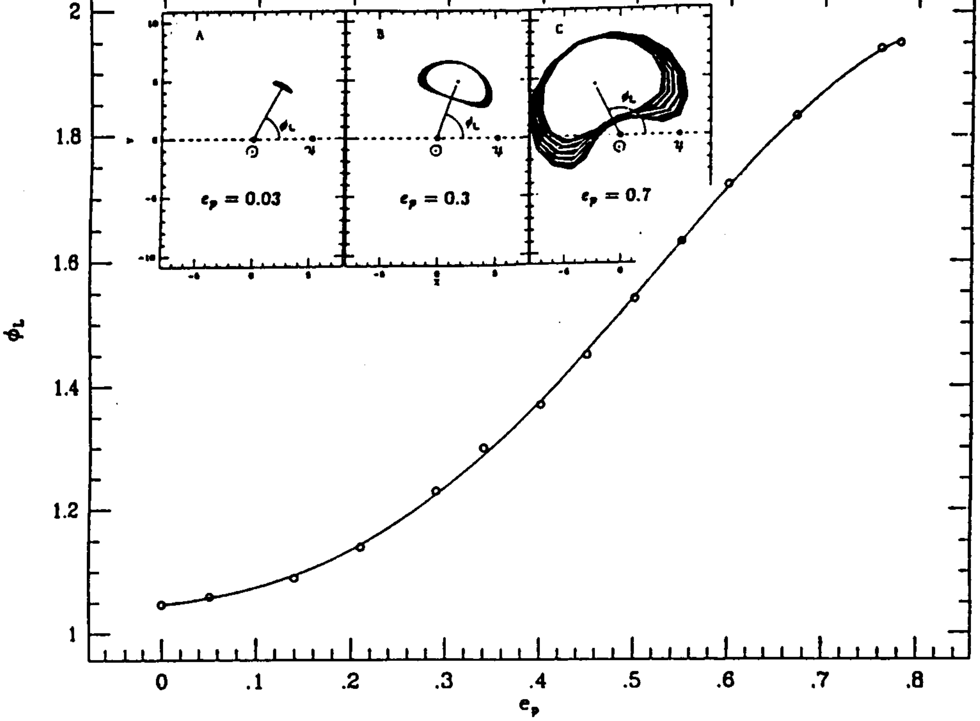

Figure 1: Position of libration point, ϕ_L, as a function of proper eccentricity, e_p.

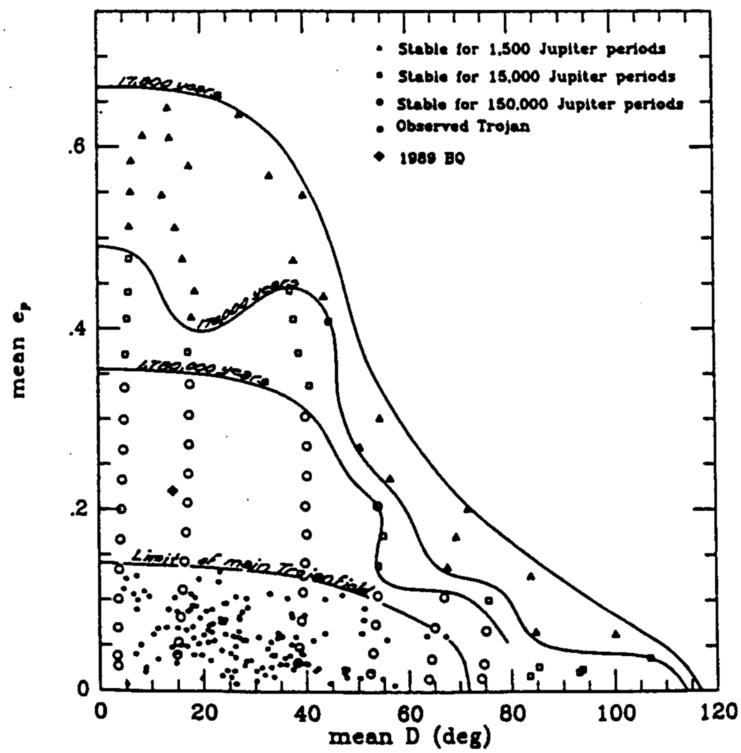

Figure 2: Map of stability of Jupiter Trojans in proper eccentricity (e_p) and libration amplitude (D) space.